Time-Based Separation Using the LWR Model

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Abstract— **This paper proposes a time-based flow model using the modified LWR highway model. This model treats each aircraft as a continuum in the flow. Instead of using density distribution, a distribution of air collision probability, the danger value distribution (DVD), is used to represent each aircraft. The closer the distance to the aircraft, the higher the danger value is. Since an aircraft passes through wider space under faster speed in a fixed time period, the faster the aircraft the wider the danger value distribution will be. This concept is the same as the surveillance zone of Traffic Collision Avoidance System (TCAS). Using the proposed flow model, danger value distribution propagates on the airway in coordination with the velocity profile of each aircraft while maintaining the characteristics of the danger level of collision. Collision can then be easily detected by the peak value of the overlap of each aircraft's danger value distribution even if each aircraft has different velocity on the airway. Once the collision is predicted, the flow should be managed by adjusting the velocity of aircraft on the airway. Hence this model can be used as a tool to optimize the air traffic in the manner of time-based separation.**

Keywords: **Time-based separation, LWR model, Traffic flow control**

I. INTRODUCTION

Nowadays, people used to travel around the world by aircraft. The rapid growth of the aviation industry causes heavier air traffic than years ago. Airways get busier and so do airports. Because of heavy air traffic, flight safety and air traffic management become more and more important. Since it is convenient to give pilots command by distance separation from radar display. Air traffic controllers (ATCs) make distance-based separation to aircraft. However, the wind speed directly affects true speed of aircraft. Strong head wind resists aircraft, thus aircraft may reduce velocity. There would be more time for aircraft to pass through the same distance under this condition. The lost velocity causes the waste of airway space. On the contrary, there will be no effect due to wind speed when aircraft are under time-based separation.

Time-based separation is not a new concept. In [1] Nicholan showed that time-based separation improves runway capacity. Distance-based separation will be affected by strong head wind during approach phase. Hence the better way to separate aircraft is time-based separation. The 3 nautical miles separation for approach phase defined by ICAO may increase the distance between leading aircraft and the following one under strong head wing, thus runway capacity will be wasted. Time-based separation is to avoid the vortex caused by the aircraft in the front. Time-based separation can also improve the use of runway capacity. With time-based separation pilots also have fixed time interval to make response. Response time will not decrease for higher velocity and that increases flight safety. Most pilots operate aircraft with the most efficient and fuel-saving velocity during the cruising phase. Management of aircraft velocity and adjustment of aircraft order is what ATCs do, to keep flight safety and mitigate air traffic. However, during the approach phase the aircraft velocity is continuously changing, and the distance of separation need to be adjusted by experienced ATCs. For example, the separation distance will be increased as a buffer during approach phase for the slowing down of the leading aircraft. The distance is increased to make sure that there are at least 3 nautical miles separation between aircraft. ATCs deal with aircraft during approach phase one by one with the same method. Quality of this work depends on the experience of ATCs. Sometimes inexperienced ATCs could get space wasted. This paper develops an air traffic flow model contains time-based separation characteristic to adjust time-based separation automatically, and that could ease the work strain of ATCs.

We use the Eulerian model [2], to construct the air traffic flow model. The goal is to control the overall air traffic flow on a single airway. Three different modes of Eulerian model have been showed in [5]. The PDE mode is better to reflect the continuous changing of traffic flow and the computation efficiency is better. The Langrangian model may cause some NP-hard problem [2], [3] is not suitable in our case.

Aircraft can been modeled as average density distributions on a portion of airways [3] [4]. The space occupied by an aircraft is centered at the aircraft position within some reference length. An aircraft is represented within the reference length of the airway with an average density. The density value is selected such that the summation of density value within the reference length of the airway will be 1 for an aircraft. In other words, an aircraft has been transformed into average density value on the airway within the reference length. And the reference length is picked

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in order to predict aircraft count correctly on airway. The reference length is just like the mean-free-path of aircraft on the airway. Crowded traffic can be observed from high density value. However, this kind of density distribution cannot provide the related position between aircraft. We only know the crowded traffic from the high density value. Which may be caused by the low velocity part of the airway. Low velocity part would cause distance loss between the leading aircraft and trailing ones, thus the density value goes high though they have the same time-based separation. Nowadays, distance separations between aircraft are 3 nautical miles during the approach phase and 5 nautical miles during the cruising phase. Distance-based separation can be easily operated through radar display, but that might cause airway capacity wasted. If time-based separation has been used, airway capacity can be more flexibly used.

The recently used collision avoidance system is Traffic Collision Avoidance System (TCAS), which is a time-based tool. In the TCAS, 25 seconds, 40 seconds, and 60 seconds surveillance zones are set [6] to cover the air space the aircraft might pass through within the specific time period. Thus surveillance zone varies with aircraft velocity. The faster the velocity the bigger the surveillance zone is. Surveillance boundary is the distance of specific time-to-go. Intruders will be alarmed with different warning levels according to the surveillance zones. With this concept, we are going to construct a flow model, in which the surveillance zone can be identified. A pulse centered at the aircraft, which represents a continuous surveillance zone is proposed, and the value of the pulse shows the warning level.

II. INTRODUCE LWR PDE

This section introduces LWR PDE of the airway [2]–[4]. Considering a portion of airway with length L, and the coordinate $x \in [0, L]$. The number of aircraft in the segment $[0, x]$ at time t is $u(x, t)$. Thus, $u(L, t)$ represents the total number of aircraft on the airway at time t. Assuming a static mean velocity profile $v(x)$ is defined on $[0, L]$, and the motion of an aircraft is described by the dynamical system $\dot{x} = v(x)$. Applying the conservation of mass to a control volume comprised between positions x and $x+h$, and letting h tend to 0, one easily finds the following relation between the spatial and temporal derivatives of $u(x, t)$:

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} + v(x)\dfrac{\partial u(x,t)}{\partial x} = q^{in}(t) & x, t \in (0, L)(0, T] \\ u(x, 0) = u_0(x) & x \in (0, L) \\ u(0, t) = 0 & t \in (0, T). \end{cases}$$

This PDE first appears in highway system [3] [4]. Now we derive a modified version of the LWR PDE on the airway.

III. AIRCRAFTS ON THE AIRWAY

Description of the position of an aircraft on the airway is another thing that we have to consider. If we set 1 for an aircraft on the position it is, then there will be a lot of spikes on the airway. This kind of delta function not only is discontinuous but also causes non-necessary numerical problems. Therefore, we represent an aircraft as a density distribution. For each aircraft, define the density function $\chi[a, b](x)$ as

$$\chi[a, b](x) = \begin{cases} 1 & \text{for } x \in [a, b] \\ 0 & \text{otherwise} \end{cases}$$

Here we take a distance centered with the real position of the aircraft as shown in Figure 1, and make a box covering the aircraft. We set the value of 1 within the box, and 0 outside the box [2]–[4]. The density is then

$$\rho(x_i, t) = \sum_{\text{aircrafts on the airway}} \frac{\chi[x_i - L_{ref}, x_i + L_{ref}](x_i)}{2L_{ref}}.$$

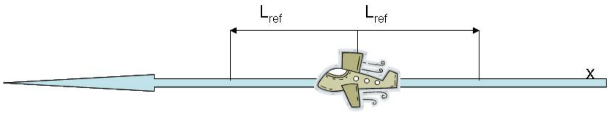

Fig. 1. The concept of how to define the density distribution of an aircraft

The number of aircraft can be obtained by

$$u(x, t) = \int_0^x \rho(y, t)dy.$$

Using ρ, the LWR PDE becomes [3]

$$\begin{cases} \dfrac{\partial \rho(x,t)}{\partial t} + v(x)\dfrac{\partial \rho(x,t)}{\partial x} = 0 & x, t \in (0, L)(0, T] \\ \rho(x, 0) = \rho_0(x) & x \in (0, L) \\ \rho(0, t) = q^{in}(t) & t \in (0, T), \end{cases}$$

where $\rho_0(x)$ means the initial density distribution on the airway, and $q^{in}(t)$ represents the incoming flow density.

In describing aircraft on the airway system, we adopt the idea of the density concept. Instead of using the characteristic function, we use a normal distribution centered at the real position of the aircraft. This kind of distribution emphasizes the real position of the aircraft.

IV. NUMERICAL SCHEME

In [5], three modes of Eulerian flow model are compared. Our PDE uses Lax Wendroff scheme [5] to determine the movement of aircraft on the airway. Lax Wendroff scheme is a scheme based on the second order Taylor expansion.

Consider the airway $x \in [0, L]$ and time $t \in [0, T]$. Let M and N divide the airway and time into M and N sectors. Hence we have x_i and t_j as

$$\begin{cases} x_i = \dfrac{iL}{M}, & 0 \le i \le M \\ t_j = \dfrac{jT}{N}, & 0 \le j \le N. \end{cases}$$

Assume that $u(x,t)$ is the solution to the LWR PDE, we can easily derive

$$\begin{cases} u_t(x,t) = -v(x)u_x(x,t) - v'(x)u(x,t) \\ u_{tt}(x,t) = -v(x)u_{xt}(x,t) - v'(x)u_t(x,t). \end{cases}$$

Then the spatial difference to u_t becomes

$$\frac{\partial u_t(x,t)}{\partial x} = -v(x)u_{xx}(x,t) - 2v'(x)u_x(x,t) - v''(x)u(x,t).$$

Using the second order Taylor expiation, we have

$$u(x,t_{j+1}) = u(x,t_j) + (\Delta t)u_t(x,t_j) + \frac{(\Delta t)^2}{2}u_{tt}(x,t_j).$$

Apply the first order and the second order mid-centered spatial differences:

$$\begin{aligned} u_x(x,t) &\leftrightarrow \frac{u_{i+1}^j - u_{i-1}^j}{2\Delta x} \\ u_{xx}(x,t) &\leftrightarrow \frac{u_{i+1}^j - 2u_i^j + u_{i-1}^j}{\Delta x^2}. \end{aligned}$$

Finally, the Lax Wendroff scheme solution is

$$\begin{aligned} u_i^{j+1} =& \{1 - (\Delta t)v'(x_i) + \frac{(\Delta t)^2}{2}[v'(x_i)]^2\}u_i^j \\ &+ \frac{\Delta t}{2\Delta x}v(x_i)[\frac{3}{2}(\Delta t)v'(x_i - 1)](u_{i+1}^j - u_{i-1}^j) \\ &+ \frac{1}{2}(\frac{\Delta t}{\Delta x})^2 v^2(x_i)(u_{i+1}^j - 2u_i^j + u_{i-1}^j). \end{aligned}$$

Here we use the ETMS data from [3]. The velocity of each aircraft is extracted from ETMS, where the aircrafts are bound to Chicago (ORD) from the east coast. Within the airways bounded to Chicago, pick up three continued links, link 1, link 4, and link 5, to simulate using the Lax Wendroff scheme. The average velocity on each position shown in Figure 2 forms the velocity profile of these links. When aircraft is in cruise phase, the slope is almost flat.

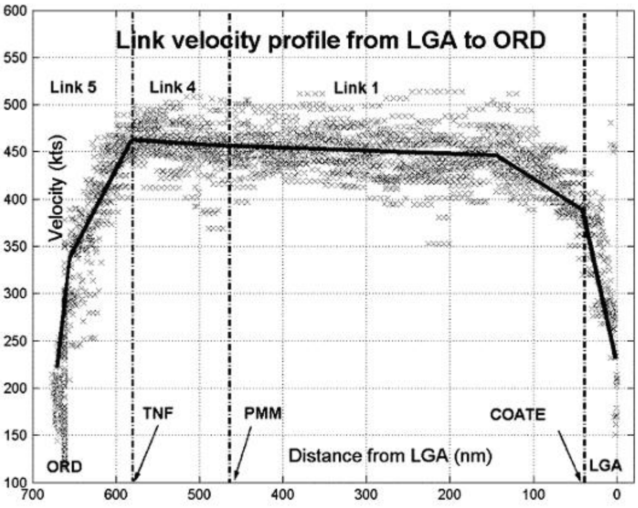

Fig. 2. ETMS data [3]

Figure 3 shows the Lax scheme result, each pulse presents the instant the aircraft traveling under the velocity profile. And the summation of each pulse should be 1.

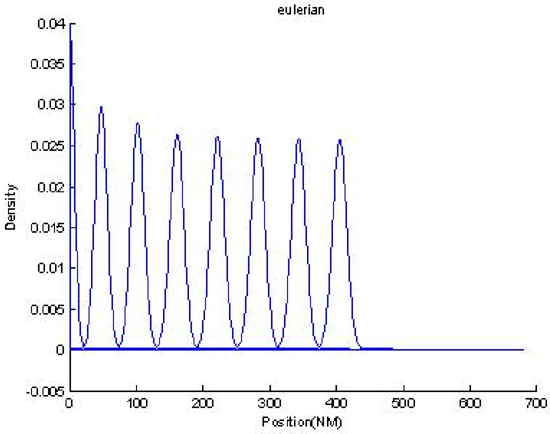

Fig. 3. Lax Wendroff scheme

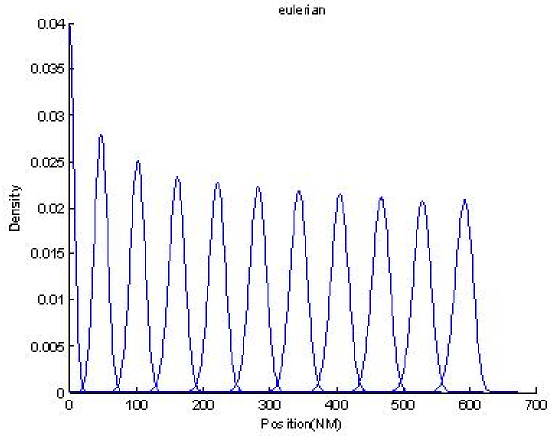

Fig. 4. Left-centered scheme

Figure 4 shows the left-centered scheme. We can see the peak value is getting lower while the pulse is propagating with the increasing velocity from both Figure 3 and Figure 4. But the summation remains the same. It makes sense since fluid spreads when the velocity is getting higher. Bayen's method is to detect crowded traffic by watching the density revolution. High density part means traffic jam. However, this phenomenon may be caused by low velocity part of the airway or the presence of too many aircraft.

V. SEPARATION DETECTION BY MODIFIED MODEL

Note that our goal is not only to determine the capacity of the airway but also to make sure if any illegal separation happens. What a controller really does in making separations is to ensure that the distance between aircraft is no less than the ICAO standards. Time-based separation separates aircraft by allowing aircraft five or few minutes period traveling distance between aircraft. And so does TCAS. The sensitivity level of TA alarm and RA alarm are defined by time but not directly defined by distance [6]. That means our model should show the section affected by an aircraft in a specified time period. And that is not only spatial but also temporal. Here, we set the normal distribution as a danger value distribution (DVD). The variance of the surveillance zone is computed using a fixed time period times the speed of the aircraft. The closer to the aircraft the higher danger value should be. In the TCAS system, there is a tau area which is a specific "protective area" around the aircraft. When an intruder aircraft enters the tau area, TCAS triggers an alarm. The threshold of the tau area is defined by time. This concept is based on time-to-go rather than distance-to-go, to the closet point of approach (CPA). Figure 5 shows how the ranges of TA (Traffic Advisory) and RA (Resolution Advisory) are defined.These ranges are similar to bubbles centered at the aircraft encircling the region by the time the aircraft would arrive. The size of the bubble depends on the velocity of the aircraft. Faster aircraft have larger bubbles. For two aircraft on the same airway, the danger value adds up when the two aircraft get close to each other. We can identify if there is a near miss by the danger value.

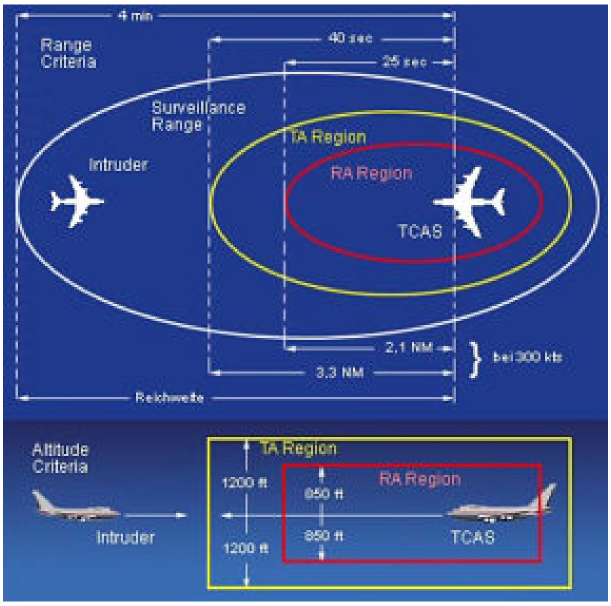

Fig. 5. TA and RA alarm region [7]

Here we introduce a time related coordinate, in which the velocity is constant. Then the propagation of pulse depends only on time. Let y be a function of $v(x)$, such that $v(y)$ is constant, i.e. the velocity of aircraft is constant in y coordinate. From

$$v(y) = \frac{dy}{dt}$$

we have

$$v(y) = \frac{dy}{dx}v(x) = c.$$

Then

$$v(x) = c\frac{dx}{dy}.$$

Now, let the flow be conservative in the y coordinate. We have

$$\frac{d(\rho(y,t)v(y))}{dt} = 0.$$

Then

$$\rho_t(y,t)v(t) + \rho_y(y,t)v(y)v(y) + \rho(y,t)v_y(y)v(y) = 0.$$

The time derivation of ρ in the y coordinate will be

$$\rho_t = -u_y(y,t)v(y) - u(y,t)v_y(y).$$

From $v(y) = c$, we have $v_y(y) = 0$. Then

$$\begin{aligned}\rho_t &= -\rho_y(y,t)v(y)\\ &= -\rho_y\frac{dy}{dx}v(x)\\ &= -\frac{\partial\rho}{\partial x}v(x)\\ &= -\rho_x(x,t)v(x).\end{aligned}$$

Similarly, we get the second order time derivation of ρ

$$\rho_{tt} = -\rho_{xt}(x,t)v(x) - \rho_x(x,t)v_t(x)$$

where

$$\rho_{xt} = -v(x)\rho_{xx}(x,t) - v'(x)\rho_x(x,t).$$

Using Lax Wendroff scheme, substitute the above to the second order Taylor expansion to get

$$\rho(x,t_{j+1}) = \rho(x,t_j) + \rho_t(x,t_j)(\Delta t) + \rho_{tt}(x,t_j)(\Delta t)^2/2.$$

The DVD of the next time instance will be

$$\begin{aligned}\rho(x,t_{j+1}) = {}&\rho(x,t_j) + (-v(x))\rho_x(x,t)(\Delta t)\\ &+ (-v(x))(-v(x)\rho_{xx}(x,t) - v'(x)\rho_x(x,t))(\Delta t)^2/2.\end{aligned}$$

This is the solution to the modified LWR PDE, which is a velocity related flow model. And it is used to detect time-based separation.

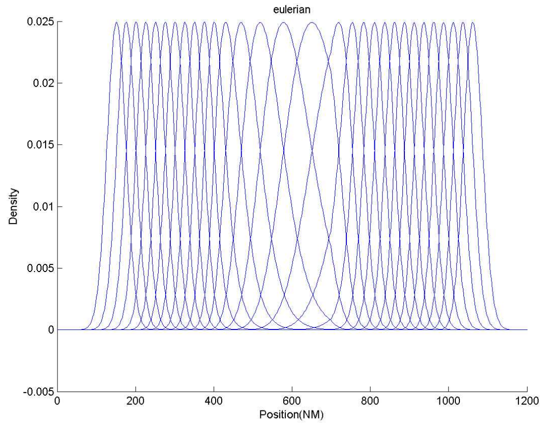

Fig. 6. Width change with velocity clearly with modified velocity profile

Figure 6 shows the simulation result with a virtual velocity profile, in which the velocity increases at beginning and decreases to the initial setting. It is easy to figure out the width of the pulse changes with velocity in Figure 6. Note that the width of the pulse changes with velocity and the peak value maintains the same when the velocity keeps changing. It can be seen that the peak value is independent of velocity. The width of the pulse gets bigger while the velocity is increasing and gets smaller while the aircraft is slowing down.

We simulate two aircraft as two equal peak value pulses with different velocity profiles. Figure 7 to Figure 10 show the simulation results at different time steps. The dashed line and the dotted line represents DVDs of two aircraft, while the solid line represents the overlapped DVD. The closer the two aircraft, the higher the peak value of the solid line is. The peak value of the solid pulse is doubled while the other two pulses are overlapped.

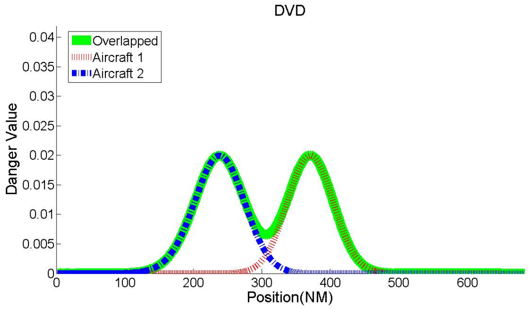

Fig. 7. t=200 seconds

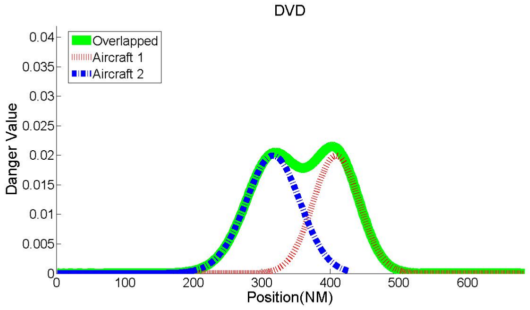

Fig. 8. t=300 seconds

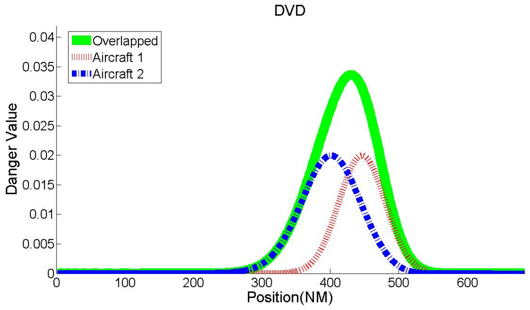

Fig. 9. t=400 seconds

VI. VELOCITY ADJUSTMENT

Since the danger value at time $t+1$ can be derived form the previous time step. We re-write the solution from Lax

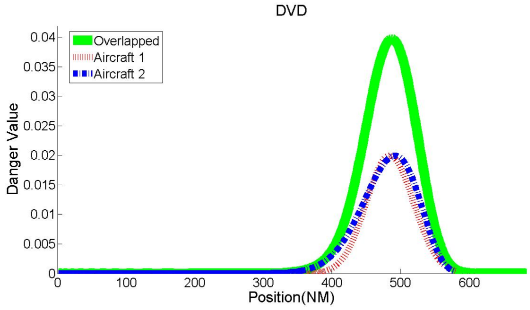

Fig. 10. t=500 seconds

scheme as a state function between two time steps with a transitional function. The solution in the vector form then be written as

$$U(t+1) = U(t) + diag(-V)U_x(t)(\Delta t)$$
$$+ diag(-V)[diag(-V)U_{xx}(t) + diag(-V')U_x(t)](\Delta t)^2/2$$

where $U(t)$ means danger value overall positions at time t and V means the velocity profile. And

$$\begin{cases} U_x &= D_1 U \\ U_{xx} &= D_2 U \\ V' &= D_1 V \end{cases}$$

where D_1 and D_2 are $M \times M$ matrices defined as

$$D_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{1}{2\Delta x}$$

$$D_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{1}{(\Delta x)_2}$$

We have the relation between time t and time $t+1$ can be written as $U(t+1) = AU(t)$ where

$$A = I + (\Delta(t))diag(-V)D_1 + diag(V)^2 D_2$$
$$+ diag(V)diag(D_1 V)D_1 \frac{(\Delta t)^2}{2}. \quad (1)$$

This model not only is capable of detecting if there is enough time-based separation, but also can be used as an optimization tool. Since DVD changes with velocity profile, a cost function that maximizes the summation of DVD on airway can be used to make the most use of the airway. Other constraints are the acceptable maximal and minimal velocity of aircraft on the airway. The key constraint is that peak value of the overlapped pulse on airway is not allowed to exceed the acceptable value. This constraint keeps enough time separation between aircraft.

Hence the optimization problem can be formulated as

$$\begin{aligned} \max \quad & H(V)=\Sigma U(t+1) \\ \text{s.t.} \quad & V \geq V_{min} \\ & V \leq V_{max} \\ & V' \leq V'_{max} \\ & U \leq U_{max}-U_{online} \end{aligned} \tag{2}$$

In which, the U_{max} is the maximal acceptable danger value and the U_{online} represents the aircraft is already on the airway.The cost function $H(V)$ can be written into the standard quadratic form as follows. From (1) and (2) the original cost function is

$$\begin{aligned} H(V) =& \Sigma(U(t)) + U_x(-V)(\Delta(t)) \\ & + V^T[diag(U_{xx}) + diag(U_x)D_1]V(\frac{(\Delta(t))^2}{2}) \end{aligned} \tag{3}$$

Note that the term $\Sigma(U(t))$ can be dropped since it is the DVD at time t which is a known value. From (3), let

$$\begin{cases} C = -(\Delta t)D_1U^T \\ Q = [diag(U_{xx}) + diag(U_x)D_1]\frac{(\Delta(t))^2}{2} \end{cases}$$

Then we have $H(V) = -(CV + V^TQV)$. Dropping the last constraint in (2), the optimal velocity profile that maximizes the DVD can be obtained by solving the following Quadratic Programming problem:

$$\begin{aligned} \min \quad & H(V) = -(CV + V^TQV) \\ \text{s.t.} \quad & V \geq V_{min} \\ & V \leq V_{max} \\ & V' \leq V'_{max} \end{aligned}$$

Figure 11 to Figure 13 show the results of maximizing DVD. Note that a faster velocity profile is obtained since with higher velocity profile the DVD expands and the airway is better utilized. This simple simulation shows how we can adjust the velocity of an aircraft through an optimization process.

VII. CONCLUSION

A congestion detection scheme is proposed. Instead of using density flow, this paper develops a DVD flow model, in which the DVD is a time related pulse. DVD is used to identify the portion on the airway using a time-based manner. The overlap of DVD then can be used to detect congestion. Combined with time-based separation, a velocity adjustment tool may be developed. With well controlled danger value, the time-based separation tool can be used to

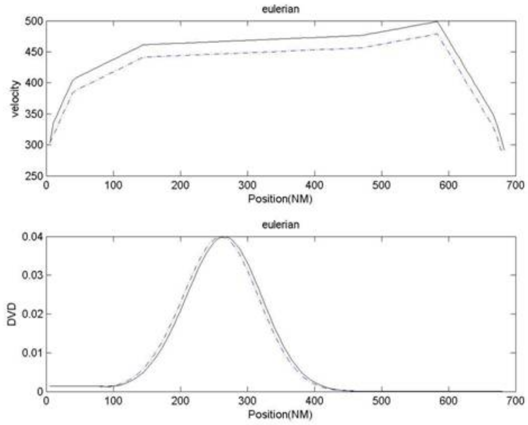

Fig. 11. Dotted line is the original setting. Solid line is the adjusted velocity profile. The figure below shows the aircraft with adjusted velocity profile goes faster.

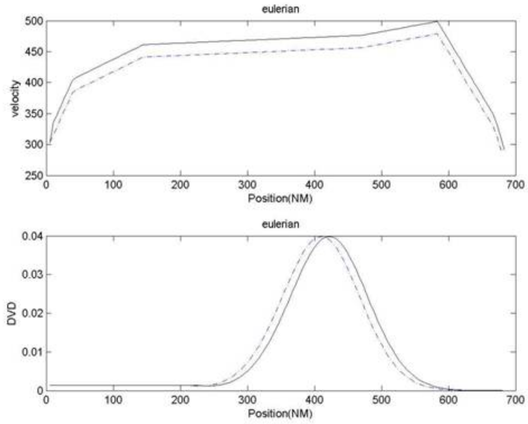

Fig. 12. Dotted line is the original setting. Solid line is the adjusted velocity profile. The figure below shows the aircraft with adjusted velocity profile goes faster.

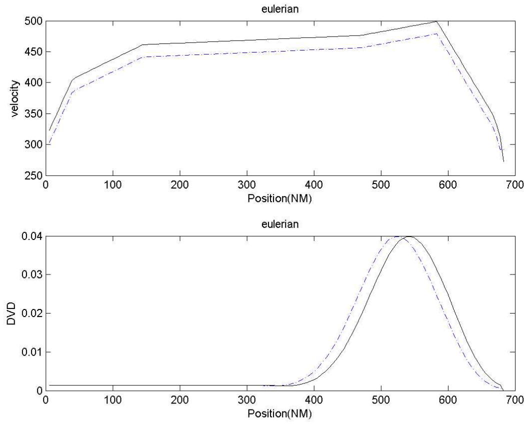

Fig. 13. Dotted line is the original setting. Solid line is the fixed one. The figure below shows the aircraft with fixed velocity profile goes faster.

ease the work stress of ATCs. The maximal danger value constraint is not applied yet. Adding this constraint makes the optimization problem a quadratically constrained quadratic programming(QCQP) which will be considered in the future.

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